The work on transcendental curves and equations in rectangular coordinates is more extensive than in the old book and has been made into a separate chapter. It includes the graphical solution of transcendental equations, and some tables to lighten the work of computation. The authors say truly in the preface that a student loses interest in a function if he cannot calculate rapidly its numerical values. It is for this reason that they have put in the first chapter tables of squares and cubes, square and cube roots, and three-place tables of logarithms and trigonometric functions.

The book is considerably larger than the *Introduction to Analytic Geometry* by the same authors.

W. H. BUSSEY.

Mathematical Recreations and Essays. Fifth Edition. By W. W. ROUSE BALL. Macmillan and Co., London, 1911. xvi+492 pages.

A great deal of new matter has been added to this interesting book since it was first published in 1892. The fifth edition contains almost 250 pages more than the first and about 100 pages more than the fourth. The work on "Kirkman's School-Girls Problem" has been enlarged and made into a separate chapter. There is a paragraph on the same problem as proposed independently by J. Steiner in a somewhat more general form. There is a new chapter of 20 pages on "The Parallel Postulate," and one of 6 pages on the "Insolubility of the Algebraic Quintic." Those who amused themselves in their youth by making figures known as *Cat's Cradles* by twisting on the hands a loop of string will be interested in the new chapter on "String Figures." The subject is more extensive than most people think. The chapter is 32 pages long and is not supposed to be a complete discussion. It is only indirectly connected with mathematics. The author explains the presence of it and the older chapters on "Astrology" and "Ciphers" by saying that he deliberately gave the book a title which would allow him a free hand to write what he liked.

The parts of the book which are not new have been revised. In the chapter on mechanical recreations, after a discussion of the cut on a tennis ball and the spin on a cricket ball, the author has put in a paragraph on the flight of golf balls. In the chapter on matter and ether theories, he has added a page on the principle of relativity. These are typical instances of the way in which the book has been brought up to date.

W. H. BUSSEY.

An Elementary Treatise on Cross-Ratio Geometry, with Historical Notes. By Rev. JOHN J. MILNE. Cambridge University Press, 1911. xxiii+288 pp.

It is well known that our literature on secondary mathematics is too limited. The ambitious teacher of secondary mathematics, who reads English only, does not possess as good facilities for broadening his knowledge as do his German and French colleagues. Recently there has been considerable improvement along this line, and the volume before us is another step in the right direction.

The author is a classical scholar as well as a mathematician, and he has enriched his work by many historical notes which tend to make it much more attractive and helpful to teachers of mathematics. For the most part these notes are not reproduced from well-known histories, as is too often the case, but they generally bear the impress of originality and of independent study. This feature enhances greatly their value and their attractiveness. The same impression is gained in regard to other parts of the work, making the book valuable to the scholar as well as to the student who may approach it without a knowledge even of the meaning of cross-ratio.

Many readers might have preferred the term *Anharmonic Ratio* instead of *Cross-Ratio* in the title, as the former is more widely used. For instance, it is used almost exclusively in France, and it is in common use in Germany, Italy, and in the English-speaking countries. In view of these facts, we may perhaps not agree with the footnote on page 2, which states that the term cross-ratio "is now generally adopted," unless this reference is supposed to apply to Great Britain only. The term anharmonic ratio is not free from objection, since it is sometimes used in two senses—as including or as excluding harmonic ratio.

In these days of emphasis on things which are closely related to the larger modern mathematical theories, it becomes especially interesting to meet a work in which the emphasis is placed on closer contact with the old Greek mathematicians, and with their methods. It is true that even in the present work attention is frequently directed to the great improvements which have been made regarding methods employed by the Greeks in the use of anharmonic ratios, but these improvements are generally in line with Greek methods.

This close contact with the spirit of the Greeks has its disadvantages. For instance, the treatment of the matter on pages 5 and 9 would have gained considerably, both in clearness and in generality, if elementary notions of groups had been introduced; showing that we are dealing here with the operations of subtracting from unity and of dividing unity, and that the successive steps of this kind give rise to six distinct operations, which constitute a very common group. It might also have been observed that these operations are special cases of more general operations which give rise to the same group.[1]

By exhibiting such contact and by putting more emphasis on the underlying abstract notions it is not only possible to make the presentation more interesting for the mature student but it is also possible to prepare the way for further advances. The days of the special geometric methods of the Greeks are passing, and the more powerful and more comprehensive methods of modern mathematics are rightly coming more and more to the front. It is, however, true that the consistent use of such a fundamental concept as that of anharmonic ratio is a great step in advance of the older and more special geometric methods of the Greeks.

Almost half of the book, Chapters I–X, is devoted exclusively to a treatment

[1] Cf. "Groups of Subtraction and Division," *Quarterly Journal of Mathematics*, vol. 37 (1906), p. 80.

of the point and a straight line, and in this part the reader is not assumed to have any knowledge of geometry beyond the fundamental properties of similar triangles and ratio. The teacher of elementary geometry will therefore find no trouble in reading this part, and he cannot fail to derive from such a reading many new points of view and a deeper sense of the richness of geometry and the beauty of geometric results.

The second part of the book, beginning with Chapter XI, page 130, is devoted to a study of conic sections. A few elementary theorems in geometrical conics are here assumed, as the author believes that time can be saved by getting a working knowledge of the elements of conic sections from the ordinary textbooks before the student is introduced to the theorems of Pascal, Brianchon, Desargues, and others. Figures are given with almost all the theorems and many full solutions are worked out in the text.

Very little use is made of projection and the principle of duality is avoided, as the author desired to exhibit the use of the theory of anharmonic ratio by the direct demonstration of correlative theorems. The book is provided with a somewhat full table of contents, and also with a good index. A number of blank pages for notes are found at the end of the volume. As a whole, the book can be regarded as a thoroughly good piece of work and one which should be in the hands of all teachers of elementary geometry.

G. A. Miller.

PROBLEMS AND QUESTIONS.

B. F. Finkel, Chairman of the Committee.

PROBLEMS FOR SOLUTION.

ALGEBRA.

383. Proposed by E. B. ESCOTT, Ann Arbor, Mich.

Find accurately to 6 decimals, $1.000000854^{1283451}$.

384. Proposed by H. C. FEEMSTER, York, Neb.

A man addressed n envelopes and wrote n checks in payment of n bills. Show that the number of ways of enclosing within each envelope one bill and one check in such a manner that in no instance all the enclosures shall be correct is

$$n!\left\{n!-\frac{(n-1)!}{1!}+\frac{(n-2)!}{2!}-\cdots+(-1)^n\frac{0!}{n!}\right\},$$

taking $0! = 0$.

385. Proposed by J. F. LAWRENCE, Stillwater, Okla.

Show that, if p is prime and > 3, $(2p)! - 2(p)!(p)!$ is divisible by p^5.

386. Proposed by ELMER SCHUYLER, Brooklyn, N. Y.

Given the sequence, $\frac{1}{2}, \frac{1}{3}, \frac{2}{3}, \frac{1}{4}, \frac{3}{4}, \frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}, \frac{1}{6}, \frac{5}{6}, \frac{1}{7}, \cdots, u_1, u_2, u_3, \cdots$. Show that

$$\left.\frac{\sum_1^n u_i}{n}\right]_{n\doteq\infty}=\frac{1}{2}.$$